Double semi-open sets with respect to a double grill

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Abstract. **The aim of this paper is to introduce double semi open sets with respect to a grill $\mathcal{D}_g$defined by double sets, we name it as $\mathcal{D}_g$semi open set and study some of its properties**

***Keywords*—double set, double grill, double semi open set, $\mathcal{D}_g$-semi open.**

1. Introduction and Preliminaries

Intuitionistic sets and intuitionistic points were introduced by D.Coker [2] in 1996. This concept was originated from the study of Zadeh [9] who introduced intuitionistic fuzzy set in the year 1965. Intuitionistic set concept is the discrete form of intuitionistic fuzzy set and it is also one of several ways of introducing vagueness in mathematical objects. In 2000, Coker [3] also introduced the concept of intuitionistic topological spaces with intutionistic sets, and investigated basic properties of continuous functions and compactness. In this paper, we follow the suggestion of Garcia and Rodabaugh [5] and we adopt the term " double set" for the intuitionistic set and "double topology" for the intuitionistic topology of Coker.

In continuation of Coker's study , GnanamballIlango and Girija [6] have given some results on intuitionistic semi open sets. The aim of this paper is to introduce double semi-open sets with respect to double grill, introduce by Rajeswari in 2017.

Definition 1.1. [2] Let X be a nonempty fixed set. A double set (DS for short) A is an object having the form A = $<$ X, A^1, $A^2$$>$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of members of A, while A^2 is called the set of non members of A. The collection of set of all double subsets of the set X is denoted as DSP(X).

Definition 1.2. [2] Let X be a nonempty set. A = $<$ X, A^1, $A^2$$>$ and B= $<$ X, B^1, $B^2$$>$be a double sets on X and let $\{A_i : i \in J\}$ be an arbitrary family of double sets in X, where A^i= $<$ X, A_i^1, $A_i^2$$>$. Then
(a) $A \subseteq B$ if and only if $A^1 \subseteq B^1$and $B^2 \subseteq A^2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $\cup A_i$= $<$ X, $\cup A_i^1$, $\cap A_i^2$$>$.
(d) $\cap A_i$ = $<$X, $\cap A_i^1$, $\cup A_i^2$$>$.
(e)$\tilde{X}$ = $<$ X, X, $\emptyset$$>$.
(f) $\tilde{\emptyset}$ = $<$X, $\emptyset$, X$>$.
(g) A^c= $<$ X, A^2, $A^1$$>$.
(h) A – B = A $\cap$ B^c.

Definition 1.3. [2] Let X be a nonempty set and p $\in$ X be a fixed element in X. Then $\tilde{p}$ = $<$X, {p}, $\{p\}^c$$>$ is called a double point (DP for short)in X.

Definition 1.4. [3] A double topology (DT for short) on a non-emptyset X is a family τ of double sets in X satisfying the following axioms:
(T_1) $\tilde{\emptyset}, \tilde{X} \in \tau$.
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$. In this case the pair (X, τ) is called a double topological space (DTS for short) and any DS in τ is known as a double open set (DOS for short) in X.

Definition 1.5. [3] The complement A of a DOS A in a double topological space (X, τ) is called a double closed set (DCS for short) in X.

Definition 1.6. [3] Let (X, τ) be a double topological space and A = $<$ X, A^1, $A^2$$>$ be a double set in X. Then the interior and closure of A are defined by Dint(A) = $\cup${G : G is a DOS in X and G $\subseteq$ A} and DCl(A) = $\cap${ K : K is DCS in X and A $\subseteq$K}.

Definition 1.7. [7] A subcollection $\mathcal{D}_g$ (not containing the empty set) of DSP(X) is called a double grill or D-grill on X if $\mathcal{D}_g$satisfies the following conditions:
(i) A $\in \mathcal{D}_g$ and A $\subseteq$B $\subseteq$X implies B $\in \mathcal{D}_g$.
(ii) A, B $\subseteq$X and A $\cup$ B $\in \mathcal{D}_g$ implies that either A $\in \mathcal{D}_g$ or B $\in \mathcal{D}_g$.

Definition 1.8. [7] Let (X, τ) be a double topological space and $\mathcal{D}_g$be a double grill on X. We define a mapping Φ: DSP(X) $\rightarrow$ DSP(X),denoted by $\Phi_{\mathcal{D}_g}$(A, τ)(for A $\in$ DSP(X)) or $\Phi_{\mathcal{D}_g}$(A) called the operator associated with the double grill $\mathcal{D}_g$ and the double topology τ and it is defined by $\Phi_{\mathcal{D}_g}$(A) = { $\tilde{p}$ $\in$X : A $\cap$ U $\in \mathcal{D}_g$for all U $\in \tau(\tilde{p})$ }, where $\tau(\tilde{p})$ stands for the collection of all open neighbourhoods of $\tilde{p}$.

Proposition 1.9. [7] Let (X, τ) be a double topological space and $\mathcal{D}_g$ be adouble grill on X. Then for all A, B$\subseteq$X, the following conditions hold:
(i) A $\subseteq$B $\subseteq$ X implies $\Phi_{\mathcal{D}_g}$(A) $\subseteq$ $\Phi_{\mathcal{D}_g}$(B)
(ii) $\Phi_{\mathcal{D}_g}$(A $\cup$ B) = $\Phi_{\mathcal{D}_g}$(A) $\cup$ $\Phi_{\mathcal{D}_g}$(B), for any A, B $\subseteq$X.
(iii) $\Phi_{\mathcal{D}_g}$($\Phi_{\mathcal{D}_g}$(A)) $\subseteq$ $\Phi_{\mathcal{D}_g}$(A) $\subseteq$ DCl(A), for any A $\subseteq$X.
(iv) For any D-grill $\mathcal{D}_g$ on X and any A $\subseteq$ X, if A$\notin \mathcal{D}_g$ then $\Phi_{\mathcal{D}_g}$(A) = $\tilde{\emptyset}$.

Definition 1.10.[6] A double set A of a double topological space (X,τ) is called double semi open (double semi closed) if A$\subseteq$Dcl(Dint(A) (resp.Dcl(Dint(A)) $\subseteq$A). The intersection (union) of double semi closed (resp. double semi open) sets containing (resp. contained in) a set A is called the double semi

closure (resp. double semi interior) of A, and is denoted by Dscl(A) (resp. Dsint(A)).
The set of all double semi open sets in a double topological space X is denoted by DSO(X).

Definition 1.11.[8] Let $\mathcal{D}_g$ be a double grill on a double topological Space (X, τ). A double set A ⊆X is called $\mathcal{D}_g$-open if A ⊆ Dint($\Phi_{\mathcal{D}_g}$(A)). The complement of such a double set is called $\mathcal{D}_g$-closed.

2. Double Semi Open sets with respect to a Double Grill

Theorem 2.1. Let $\mathcal{D}_g$ be a double grill on X. We define a map $\psi_{\mathcal{D}_g}$: DS P(X) → DS P(X) by $\psi_{\mathcal{D}_g}$(A) = A ∪$\Phi_{\mathcal{D}_g}$(A), for all A ∈DS P(X).Then the map $\psi_{\mathcal{D}_g}$ is a kuratowski's operator.

Proof: For any non-empty set A of X, if A ∉ $\mathcal{D}_g$,then $\Phi_{\mathcal{D}_g}$(A) = $\widetilde{\emptyset}$. Since $\widetilde{\emptyset}$ ∉ $\mathcal{D}_g$ and $\psi_{\mathcal{D}_g}(\widetilde{\emptyset})$ = $\widetilde{\emptyset}$ ∪ $\Phi_{\mathcal{D}_g}(\widetilde{\emptyset})$ = $\widetilde{\emptyset}$. Obviously, A ⊆ $\psi_{\mathcal{D}_g}$(A) for all A ⊆X. Also $\psi_{\mathcal{D}_g}$(A ∪B) = (A ∪B) ∪ $\Phi_{\mathcal{D}_g}$(A ∪B) = (A ∪B)∪ $\Phi_{\mathcal{D}_g}$(A) ∪ $\Phi_{\mathcal{D}_g}$(B) = A ∪ $\Phi_{\mathcal{D}_g}$(A) ∪B ∪ $\Phi_{\mathcal{D}_g}$(B) = $\psi_{\mathcal{D}_g}$(A) ∪ $\psi_{\mathcal{D}_g}$(B). Again for any A ⊆X, $\psi_{\mathcal{D}_g}(\psi_{\mathcal{D}_g}$(A)) = $\psi_{\mathcal{D}_g}$(A ∪ $\Phi_{\mathcal{D}_g}$(A)) =A ∪ $\Phi_{\mathcal{D}_g}$(A) ∪ $\Phi_{\mathcal{D}_g}$(A ∪ $\Phi_{\mathcal{D}_g}$(A)) = A ∪ $\Phi_{\mathcal{D}_g}$(A) ∪ $\Phi_{\mathcal{D}_g}(\Phi_{\mathcal{D}_g}$(A)= A ∪ $\Phi_{\mathcal{D}_g}$(A) = $\psi_{\mathcal{D}_g}$(A).

As this operator $\psi_{\mathcal{D}_g}$ satisfies the conditions of a kuratowski's operator and using this $\psi_{\mathcal{D}_g}$ function, we can define a double topology as below.

Definition 2.2. For a given double topology(X, τ) and corresponding to a double grill $\mathcal{D}_g$,there exists a unique double topology $\tau_{\mathcal{D}_g}$ (say) on X is given by $\tau_{\mathcal{D}_g}$ = {U ∈ DSP(X): $\psi_{\mathcal{D}_g}(\widetilde{X}$ /U) = $\widetilde{X}$ /U }where for any double set A of X, $\psi_{\mathcal{D}_g}$(A) = A ∪ $\Phi_{\mathcal{D}_g}$(A) = $\tau_{\mathcal{D}_g}$-Dcl(A), where $\tau_{\mathcal{D}_g}$-Dcl(A) is the closure of the double set A with respect to the topology $\tau_{\mathcal{D}_g}$ and $\tau_{\mathcal{D}_g}$-Dcl(A) is the intersection of all $\tau_{\mathcal{D}_g}$- closed sets containing A. Also $\tau_{\mathcal{D}_g}$-Dint(A) is the union of all $\tau_{\mathcal{D}_g}$-open sets contained in A.

Definition 2.3. A double set A of a D-grill topological space (X, τ) is said to be double semi open with respect to the D-grill $\mathcal{D}_g$ or simply $\mathcal{D}_g$-semi open if A⊆ $\psi_{\mathcal{D}_g}$(Dint(A)). The complement of a $\mathcal{D}_g$- semi open set is called a $\mathcal{D}_g$- semi closed set.

Remark 2.4. Every $\mathcal{D}_g$-semi open set is double semi open. Infact, let A be $\mathcal{D}_g$- semi open in (X,τ). Then A ⊆ $\psi_{\mathcal{D}_g}$(Dint(A)) = Dint(A) ∪ $\Phi_{\mathcal{D}_g}$(Dint(A))⊆ Dint(A) ∪ (Dcl(Dint(A))= Dcl(Dint(A). Therefore A is double semi open. But the converse is not be true. This is justified in Example 2.5.

Example 2.5. Let X = {a,b,c}, τ = {$\widetilde{\emptyset}$, $\widetilde{X}$, $< X, \{b\},\{a,c\}>$, $<X,\{c\},\{a,b\}>$, $<X, \{b,c\}, \{a\}>$} be a double topology on X and $\mathcal{D}_g$ = { $< X, \{a\},\{b,c\}>$, $< X, \{a\},\{b\}>$, $< X,\{a\},\{c\}>$, $<X,\{a,b\},\{c\}>$, $<X,\{a,c\},\{b\}>$, $<X,\{a\},\emptyset>$, $<X, \{a,b\},\emptyset>$, $<X, \{a,c\},\emptyset>$, $\widetilde{X}$ } be a D-grill on X.
Take A = $< X, \{a,c\},\{b\}>$.Then Dint(A) = $< X, \{c\},\{a,b\}>$, $\Phi_{\mathcal{D}_g}$(Dint(A))= $\widetilde{\emptyset}$ and $\psi_{\mathcal{D}_g}$(Dint(A)) = $< X,\{c\},\{a,b\}>$. Therefore $< X, \{a,c\},\{b\}>$⊈ $< X, \{c\},\{a,b\}>$. Hence A is not $\mathcal{D}_g$- semi open. Also Dcl(Dint(A)) = $< X, \{a,c\},\{b\}>$. Hence A is double semi open.

Remark 2.6. The concepts of $\mathcal{D}_g$-semi open and $\mathcal{D}_g$-open sets are independent of each other.

Example 2.7. Let X = {a,b,c}, τ = { $\widetilde{\emptyset}$, $\widetilde{X}$, $< X, \{a\},\{b,c\}>$, $<X,\{b\}, \{a,c\}>$, $<X, \{a,b\}, \{c\}>$} be a double topology on X and $\mathcal{D}_g$ = { $< X, \{c\},\{a,b\}>$, $<X,\{c\},\{b\}>$, $<X, \{c\},\{a\}>$, $<X,\{a,c\},\{b\}>$, $<X,\{b,c\},\{a\}>$, $<X,\{a,c\},\emptyset>$, $<X,\{b,c\},\emptyset>$, $<X, \{c\},\emptyset>$, $\widetilde{X}$ } be a D-grill on X.
Suppose A = $< X, \{a,b\},\{c\}>$. Then $\Phi_{\mathcal{D}_g}$(A) = $\widetilde{\emptyset}$ and Dint($\Phi_{\mathcal{D}_g}$(A)) = $\widetilde{\emptyset}$, so A is not $\mathcal{D}_g$-open. Now Dint(A) = $< X, \{a,b\},\{c\}>$ and $\Phi_{\mathcal{D}_g}$(Dint(A)) = $\widetilde{\emptyset}$, so that $\psi_{\mathcal{D}_g}$(Dint(A)) = $< X, \{a,b\},\{c\}>$. Hence A is $\mathcal{D}_g$- semi open.

Example 2.8. Consider the double topological space (X, τ) where X = {a,b,c} and τ = { $\widetilde{\emptyset}$, $\widetilde{X}$, $< X, \{a\},\{b,c\}>$, $<X,\{b,c\}, \{a\}>$} be a double topology on X and $\mathcal{D}_g$ = { $< X, \{a\},\{b,c\}>$, $< X, \{c\},\{a,b\}>$, $< X, \{a\},\{b\}>$, $< X, \{a\},\{c\}>$, $< X, \{a,b\},\{c\}>$, $< X, \{a,c\},\{b\}>$, $< X, \{c\},\{a\}>$, $< X, \{c\},\{b\}>$, $< X, \{b,c\},\{a\}>$, $<X, \{a\},\emptyset>$, $<X, \{a,b\},\emptyset>$, $<X, \{a,c\},\emptyset>$, $<X, \{b,c\},\emptyset>$, $<X, \{c\},\emptyset>$, $\widetilde{X}$ } be a D-grill on X.
Suppose A = $< X, \{a,c\},\{b\}>$. Then $\Phi_{\mathcal{D}_g}$(A) =$\widetilde{X}$ and Dint($\Phi_{\mathcal{D}_g}$(A)) = $\widetilde{X}$. Hence A is $\mathcal{D}_g$-open. Again Dint(A) = $<X,\{a\},\{b,c\}>$ and $\Phi_{\mathcal{D}_g}$(Dint(A)) = $< X,\{a\},\{b,c\}>$ and $\psi_{\mathcal{D}_g}$(Dint(A)) = $< X, \{a\},\{b,c\}>$⊉ A. Hence A is not $\mathcal{D}_g$-semi open.

Theorem 2.9. Let (X, τ) be a double topological space and $\mathcal{D}_g$ be a D-grill on X. Then a double set A of X is $\mathcal{D}_g$ – semi open if and only if $\psi_{\mathcal{D}_g}$(A) = $\psi_{\mathcal{D}_g}$(Dint(A)).

Proof: Let A be $\mathcal{D}_g$-semiopen. Then A ⊆ $\psi_{\mathcal{D}_g}$(Dint(A)) implies $\psi_{\mathcal{D}_g}$(A) ⊆ $\psi_{\mathcal{D}_g}(\psi_{\mathcal{D}_g}$(Dint(A)))= $\psi_{\mathcal{D}_g}$(Dint(A)) ⊆ $\psi_{\mathcal{D}_g}$(A) implies $\psi_{\mathcal{D}_g}$(A) = $\psi_{\mathcal{D}_g}$(Dint(A)). The converse is trivial.

Theorem 2.10. Let (X, τ) be a double topological space and $\mathcal{D}_g$ be a D-grill on X. If a double set A of X is $\mathcal{D}_g$-semiclosed, then Dint($\psi_{\mathcal{D}_g}$(A)) ⊆A.

Proof: Suppose A is $\mathcal{D}_g$-semi closed. Then, $\widetilde{X}$ \ A is $\mathcal{D}_g$-semi open and hence $\widetilde{X}$ \ A ⊆ $\psi_{\mathcal{D}_g}$(Dint($\widetilde{X}$ \ A)) ⊆ Dcl(Dint($\widetilde{X}$ \ A)) = $\widetilde{X}$ \ Dint(Dcl(A)) ⊆ $\widetilde{X}$ \ Dint($\psi_{\mathcal{D}_g}$(A)) implies Dint($\psi_{\mathcal{D}_g}$(A)) ⊆ A.

Remark 2.11. The converse of Theorem 2.10 is not true. This is Justified with Example 2.12.

Example 2.12. Consider the double topological space (X, τ), where X = {a,b,c} and τ = { $\widetilde{\emptyset}$, $\widetilde{X}$, $< X, \{a\},\{b,c\}>$, $<X,\{b,c\},\{a\}>$} be a double topology on X and $\mathcal{D}_g$ = { $< X, \{a\},\{b,c\}>$, $< X, \{a\},\{b\}>$, $< X, \{a\},\{c\}>$, $< X, \{a,b\},\{c\}>$, $< X, \{a,c\},\{b\}>$, $<X, \{a\},\emptyset>$, $<X, \{a,b\},\emptyset>$, $<X, \{a,c\},\emptyset>$, $\widetilde{X}$ } be a D-grill on X.
Suppose A = $< X, \{a,c\},\{b\}>$. Then $\Phi_{\mathcal{D}_g}$(A) = $< X, \{a\},\{b,c\}>$ and $\psi_{\mathcal{D}_g}$(A) = $< X, \{a,c\},\{b\}>$. That implies Dint($\psi_{\mathcal{D}_g}$(A)) = $< X, \{a\},\{b,c\}>$. Therefore Dint($\psi_{\mathcal{D}_g}$(A)) ⊆ A.

But $\widetilde{X} \setminus A$ is not $\mathcal{D}_g$-semi open. Therefore A is not $\mathcal{D}_g$-semi closed.

Theorem 2.13. Let (X, τ) be a double topological space with a D-grill $\mathcal{D}_g$ and A be a double set of X such that $\widetilde{X} \setminus \mathrm{Dint}(\psi_{\mathcal{D}_g}(A)) = \psi_{\mathcal{D}_g}(\mathrm{Dint}(\widetilde{X} \setminus A))$. Then A is $\mathcal{D}_g$-semi closed if and only if $\mathrm{Dint}(\psi_{\mathcal{D}_g}(A)) \subseteq A$.

Proof: Let A be $\mathcal{D}_g$-semi closed. By Theorem 2.10, $\mathrm{Dint}(\psi_{\mathcal{D}_g}(A)) \subseteq A$.

Conversely, suppose that $\mathrm{Dint}(\psi_{\mathcal{D}_g}(A)) \subseteq A$. Then $\widetilde{X} \setminus A \subseteq \widetilde{X} \setminus \mathrm{Dint}(\psi_{\mathcal{D}_g}(A)) = \psi_{\mathcal{D}_g}(\mathrm{Dint}(\widetilde{X} \setminus A))$ implies $\widetilde{X} \setminus A$ is $\mathcal{D}_g$-semi open. Hence A is $\mathcal{D}_g$-semi closed.

Theorem 2.14. Let (X, τ) be a double topological space and $\mathcal{D}_g$ be a D-grill on X. Then a double set A of X is $\mathcal{D}_g$-semi open if and only if there exists a U ∈ τ such that $U \subseteq A \subseteq \psi_{\mathcal{D}_g}(U)$.

Proof: Suppose A is $\mathcal{D}_g$-semi open. Then $A \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(A))$. Take Dint(A) = U. Then $U \subseteq A \subseteq \psi_{\mathcal{D}_g}(U)$, where U ∈ τ. Conversely, let $U \subseteq A \subseteq \psi_{\mathcal{D}_g}(U)$, where U ∈ τ. Now $U \subseteq A$ implies $U \subseteq \mathrm{Dint}(A)$ and so $\psi_{\mathcal{D}_g}(U) \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(A))$ which implies $A \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(A))$. Therefore A is $\mathcal{D}_g$-semi open.

Theorem 2.15. Let (X, τ) be a double topological space and $\mathcal{D}_g$ be a double grill on X. Let A and B be double sets of X such that $A \subseteq B \subseteq \psi_{\mathcal{D}_g}(A)$. If A is $\mathcal{D}_g$-semi open, then B is also $\mathcal{D}_g$-semi open.

Proof: Let A be $\mathcal{D}_g$-semi open. Then there exists U ∈ τsuch that $U \subseteq A \subseteq \psi_{\mathcal{D}_g}(U)$ implies $U \subseteq A \subseteq B \subseteq \psi_{\mathcal{D}_g}(A) \subseteq \psi_{\mathcal{D}_g}(\psi_{\mathcal{D}_g}(U)) = \psi_{\mathcal{D}_g}(U)$.Therefore B is $\mathcal{D}_g$-semi open.

Theorem 2.16. Let (X, τ) be a double topological space and $\mathcal{D}_g$ be a D-grill on X.
(a) If $\{U_\alpha : \alpha \in J\}$ is a family of $\mathcal{D}_g$-semi open sets, then $\cup\{U_\alpha : \alpha \in J\}$is $\mathcal{D}_g$-semiopen.
(b) If $A \subseteq X$ is $\mathcal{D}_g$-semi open and U ∈ τ, then $A \cap U$ is$\mathcal{D}_g$-semiopen.

Proof: (a) Suppose $U_\alpha$is $\mathcal{D}_g$-semiopen, for each α ∈J. Then $U_\alpha \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(U_\alpha))$ for each α ∈ J implies $\cup U_\alpha \subseteq \cup \psi_{\mathcal{D}_g}(\mathrm{Dint}(U_\alpha)) \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(\cup U_\alpha))$. Therefore $\cup\{U_\alpha : \alpha \in J\}$is $\mathcal{D}_g$-semi open.
(b) Let A be $\mathcal{D}_g$-semi open and U ∈ τ. Then $A \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(A))$. Now $A \cap U \subseteq \psi_{\mathcal{D}_g}(\mathrm{Dint}(A)) \cap U = (\mathrm{Dint}(A) \cup \Phi_{\mathcal{D}_g}(\mathrm{Dint}(A)) \cap U = (\mathrm{Dint}(A) \cap U) \cup (\Phi_{\mathcal{D}_g}(\mathrm{Dint}(A)) \cap U) \subseteq \mathrm{Dint}(A \cap U) \cup \Phi_{\mathcal{D}_g}(\mathrm{Dint}(A \cap U)) = \psi_{\mathcal{D}_g}(\mathrm{Dint}(A \cap U))$. Therefore $A \cap U$ is $\mathcal{D}_g$-semi open.

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