On Anti-Fuzzy Bi-ideals In Near Subtraction Semigroups

P. Annamalai Selvi[1], R. Sumitha[2], Dr. S. Jayalakshmi[3]

[1,2]Research Scholar, Sri Parasakthi College for Women, Courtallam. India.
[3]Associate Professor, Sri Parasakthi College for Women, Courtallam. India.

Abstract: **In this paper we introduce the notation of anti-fuzzy bi-ideals innear subtraction semigroup and give some characterizations of fuzzy bi-ideals in near subtraction semigroup. We establish that every fuzzy right ideal,fuzzy ideals are fuzzy bi-ideals of a near- subtraction semigroup. But the converse is not necessarily true as shown by anexample.**

Key words: **Near subtraction semigroups, fuzzy bi-ideal, anti fuzzy bi-ideal.**

1. Introduction

B.M.Schein [1] considered systems of the form (X; o;/), where X is a set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of [1]).Y.B.Jun et al[2] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals.In[8], Y.B.Jun and H.S.Kim established the ideal generated by a set, and discussed related results. The concept of fuzzy set was first initiated by Zadeh[7]. Narayanan et al.[5] defined the concept of generalized fuzzy ideals of near-rings.Mahalakshmi et al. [3] studied the notation of bi-ideals in near subtraction semigroups. Manikandan [4] studied fuzzy bi-ideals in near-rings.

2. Preliminaries

Definition: 2.1 A nonempty set X together with binary operations "–" and is said to be subtraction algebra if it satisfies the following:

(i) $x-(y-x) = x$.
(ii) $x-(x-y) = y-(y-x)$.
(iii) $(x-y)-z = (x-z)-y$,
for every $x,y,z \in X$.

Definition:2.2 A nonempty set X together with two binary operations "–" and "•" is said to be a subtraction semigroup if it satisfies the following:

(i) (X,–) is a subtraction algebra.
(ii) (X,•) is a semigroup.
(iii) $x(y-z) = xy - xz$ and $(x-y)z = x$
for every $x, y, z \in X$.

Definition:2.3 A non empty set X together with two binary operations "–"and"•" 'is said to be a near subtraction semigroup(right) if it satisfies the following:

(i) (X, –) is a subtraction algebra.
(ii) (X, •) is a semigroup.
(iii) $(x-y)z = xz - yz$
for every $x,y,z \in X$.

It is clear that $0x = 0$, for all $x \in X$. Similarly we can define a left near- subtraction semigroup. Here after a near – subtraction semigroup means only a right near-subtraction semigroup.

Definition:2.4 A near subtraction semigroup X is said to be Zero – symmetric if $x0 = 0$ for every $x \in X$.

Definition:2.5 A nonempty subset S of a subtraction semigroup X is said to be a subalgebra of X, if $x-y \in S$, for all$x, y \in S$.

Definition:2.6 A nonempty subset S of a near - subtraction algebra X is said to be anear subtraction subsemigroupof X, if $x-y \in S, xy \in S$ for all$x,y \in X$.

Definition:2.7 Let (X, - , .) be a near – subtraction semigroup. A nonempty subset I of X is called

(i) A left ideal if I is a subalgebraof(X , –) and $xi - x(y-i) \in$ for all $x, y \in X$ and $i \in I$
(ii) A right ideal I is a subalgebraof(X , –) and $IX \subseteq I$.
(iii) An ideal of X if I is both left and right ideal of X.

Definition:2.8 A fuzzy subset μ is called fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(xi - x(y-i)) \geq \mu(i)$

(iii) $\mu(xy) \geq \mu(x)$ for all $x, y \in X$

Definition:2.9 A fuzzy subset μ of X is said to be a fuzzysubalgebra of X, if$x,y \in X$ implies $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$

Definition: 2.10 A fuzzy subalgebra μ of X is called a fuzzy bi-ideal of X if

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$
(ii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$

for all $x, y, z \in X$

Example: Let X={0,a,b,c} in which '–' and '•'are defined by

–	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	0	0	0	c

Then (X, –, .) is a near subtraction semigroup. Let $\mu: X \to [0, 1]$ be a fuzzy subset of X defined as $\mu(0) = 0.9$,$\mu(a) = 0.7$,$\mu(b) = 0.6$ and $\mu(c) = 0.4$.Then μ is a fuzzy bi-ideal of X.

3. Anti-fuzzy bi-ideals

Definition: 3.1 A fuzzy subalgebra μ of X is called a anti-fuzzy bi-ideal of X if

(i) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$

(ii) $\mu(x\,y\,z) \leq \max\{\mu(x), \mu(z)\}$

for all $x, y, z \in X$.

Example: Let X = {0,a,b,c} in which '–' and '•'are defined by

–	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Then (X, –, .) is a near subtraction semigroup. Let μ: X→[0, 1] be a fuzzy subset of X defined as $\mu(0) = 0.6$, $\mu(a) = 0.7$, $\mu(b) = \mu(c) = 0.8$.Then μ is an antifuzzy bi-ideal of X. But μ is not a fuzzy bi-ideal of X. Since $\mu(0)=\mu(b-b) \ngeq \min\{\mu(b),\mu(b)\}$.

Definition: 3.12 A family of fuzzy set

$\{\mu_i \,/\, I \in \wedge\}$ is a near-subtraction semigroup X, the union $\vee_{i\in\wedge}\mu_i$ of $\{\mu_i \,/\, i\in\wedge\}$is defined by $(\vee_{i\in\wedge}\mu_i)\,(x) = \sup\{\mu_i(x) \,/$

$i \in\wedge\}$for each $x \in X$.

Definition:3.13 A family of fuzzy set

$\{\mu_i \,/\, i \in\wedge\}$ is a near-subtraction semigroup X, the intersection $\cap_{i\in\wedge}\mu_i$ of $\{\mu_i \,/\, I \in\wedge\}$is defined by $(\cap_{i\in\wedge}\mu_i)\,(x) = \inf\{\mu_i(x) \,/\, i \in\wedge\}$for each $x \in X$.

Definition:3.14 Let f be a mapping from a set X to a set X'. Let μ and λ be fuzzy subset of X and X', respectively. Then $f(\mu)$, the image of μ under f is a subset of X' defined by

$$f(\mu) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \phi \\ \text{otherwise}, & 0 \end{cases}$$

And the pre-image of λ under f is the fuzzy subset defined by $f^{-1}(\lambda(x)) = \lambda(f(x))$, for all $x\in X$ and $f^{-1}(y) = \{x\in X \,/\, f(x) = y\}$.

Definitions: 3.15 A fuzzy bi-ideal μ of a near subtraction semigroup X is said to be normal if $\mu(0) =1$.An anti-fuzzy bi-ideal μ of a near subtraction semigroup X is said to be complete if it is normal and there exists $z \in X$ such that $\mu(z) = 0$.

Theorem:3.16

Let X be a near subtraction semigroup and μ be a fuzzy set in X. Then μ is a fuzzy bi-ideal in X iff μ^c is a anti-fuzzy bi-ideal.

For all x, y, z $\in$X.

Proof:

(i)$\mu^c(x - y) = 1 - \mu(x - y)$
$\leq 1 - \min\{\mu(x),\mu(y)\}$
$= \max\{1 - \mu(x),\ 1 - \mu(y)\}$
$= \max\{\mu^c(x), \mu^c(y)\}$

$\therefore \mu^c(x - y) \leq \max\{\mu^c(x), \mu^c(y)\}$

(ii)$\mu^c(x\,yz) = 1 - \mu(x\,yz)$
$\leq 1 - \min\{\mu(x),\mu(z)\}$
$= \max\{1 - \mu(x), 1 - \mu(z)\}$
$= \max\{\mu^c(x), \mu^c(z)\}$

$\therefore \mu^c(xyz) \leq \max\{\mu^c(x), \mu^c(z)\}$

Hence μ^c is a anti-fuzzy bi-ideal in X.

Conversely assume that μ^cisa anti-fuzzy bi-ideal in X. For all x , y, z $\in$X.

(i) $\mu(x - y) = 1 - \mu^c(x - y)$
$\geq 1 - \max\{\mu^c(x),\mu^c(y)\}$
$= \min\{1 - \mu^c(x), 1 - \mu^c(y)\}$
$= \min\{\mu(x), \mu(y)\}$

$\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii)$\mu(x\,yz) = 1 - \mu^c(x\,yz)$
$\geq 1 - \max\{\mu^c(x),\mu^c(z)\}$
$= \min\{1- \mu^c(x), 1- \mu^c(z)\}$
$= \min\{\mu(x), \mu(z)\}$

$\therefore \mu(xyz) \geq \min\{\mu(x), \mu(z)\}$

Hence $\mu$is a fuzzy bi-ideal in X.

Theorem:3.17

Let μ be a fuzzy set in a near subtraction semigroup X. Then μ is a fuzzy bi-ideal of X iff the upper level cut $U(\mu\,;t)$ of X is a bi-ideal of X for each $t \in [\mu(0), 1]$.

Proof:

Let μ is a fuzzy bi-ideal of X.Let $x, y \in U(\mu\,;t)$.Then $\mu(x) \geq t$ and $\mu(y) \geq t$.

Now, $\mu(x - y) \geq \min\{\mu(x), \mu(y)\} = t \Rightarrow \mu(x - y) \geq t$ and so $x - y \in U(\mu\,;t)$.

Hence $U(\mu\,;t)$ is a subalgebra of X.

Let $x, z \in U(\mu\,;t)$ and $y \in X$.

Then $\mu(x) \geq t$ and $\mu(z) \geq t$.

Now $\mu(xyz) \geq \min\{\mu(x), \mu(z)\} = t \Rightarrow \mu(xyz) \geq t$, and so $xyz \in U(\mu\,;t)$.

Hence $U(\mu\,;t)$ is a bi-ideal of X.

Conversely assume that $U(\mu\,;t)$ is a bi-ideal of X.To prove that μ is a fuzzy bi-ideal of X.

Suppose $\mu$is not a fuzzy bi-ideal of X. Suppose $x, y \in X$ and $\mu(x - y) < \min\{\mu(x), \mu(y)\}$.Choose t such that $\mu(x - y) < t < \min\{\mu(x), \mu(y)\}$.Then we get $x, y \in U(\mu\,;t)$.

But $x - y \notin U(\mu\,;t)$.which is a contraction.

Hence $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$.

Suppose $x, y, z \in X$.$\mu(xyz) < \min\{\mu(x), \mu(z)\}$.

Choose t such that $\mu(xyz) < t < \min\{\mu(x), \mu(z)\}$. Then we get $x, z \in U(\mu\,;t)$.

But $xyz \notin U(\mu\,;t)$.which is a contraction.

Hence $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$.

Hence μ is a fuzzy bi-ideal of X.

Theorem: 3.18

If $\{\mu_i/ \ i\in\wedge\}$ is a family of fuzzy bi-ideals of a near subtraction semigroup X.

Then $\cap_{i\in\wedge}\mu_i$is a fuzzy bi-ideal.

Proof:

Let $\{\mu_i/ \ i\in\wedge\}$ is a family of fuzzy bi-ideals of a near subtraction semigroup X.

Let x , y, z $\in$ X.

(i)$\cap_{i\in\wedge}\mu_i(x - y) = \inf\{\mu_i(x - y) \,/\, i \in\wedge\}$
$\geq \inf\{\min\{\mu_i(x), \mu_i(y)\} \,/\, i\in\wedge\}$
$= \min\{\inf\{\mu_i(x) \,/\, i \in\wedge\}, \inf\{\mu_i(y) \,/\, i \in\wedge\}\}$ $=\min\{(\wedge_{i\in\wedge}\mu_i)(x), (\wedge_{i\in\wedge}\mu_i)(y)\}$

$\therefore \bigcap_{i\in\wedge}\mu_i(x-y) \geq \min\{(\wedge_{i\in\wedge}\mu_i))(x), (\wedge_{i\in\wedge}\mu_i))(y)\}$

(ii) $\bigcap_{i\in\wedge}\mu_i(xyz) = \inf\{\mu_i(xyz) / i \in \wedge\}$

$\geq \inf\{\min\{\mu_i(x), \mu_i(z) / i \in \wedge\}$

$= \min\{\inf\{\mu_i(x) / i \in \wedge\}, \inf\{\mu_i(z) / i \in\}\}$

$= \min\{(\wedge_{i\in\wedge}\mu_i)(x), ((\wedge_{i\in\wedge}\mu_i)(z)\}$

$\therefore \bigcap_{i\in\wedge}\mu_i(xyz) \geq \min\{(\wedge_{i\in\wedge}\mu_i)(x), (\wedge_{i\in\wedge}\mu_i)(z)\}$

Hence $\therefore \bigcap_{i\in\wedge}\mu_i$ is a fuzzy bi-ideal of X.

Theorem:3.19

Let μ be a fuzzy bi ideal of a near subtraction semigroup X and μ^* be a fuzzy set in X defined by $\mu^*(x) = \mu(x) + 1 - \mu(0)$ for all $x \in X$. Then μ^* is a fuzzy bi-ideal of X containing μ.

Proof:

Let μ be a fuzzy bi ideal of a near subtraction semigroup X. For any $x, y \in X$.

(i) $\mu^*(x-y) = \mu(x-y) + 1 - \mu(0)$

$\geq \min\{\mu(x), \mu(y)\} + 1 - \mu(0)$

$= \min\{\mu(x) + 1 - \mu(0), \mu(y) + 1 - \mu(0)\}$

$= \min\{\mu^*(x), \mu^*(y)\}$

$\therefore \mu^*(x-y) \geq \min\{\mu^*(x), \mu^*(y)\}$

For any $x, y, z \in X$.

(ii) $\mu^*(xyz) = \mu(xyz) + 1 - \mu(0)$

$\geq \min\{\mu(x), \mu(z)\} + 1 - \mu(0)\}$

$= \min\{\mu(x) + 1 - \mu(0), \mu(z) + 1 - \mu(0)\}$

$= \min\{\mu^*(x), \mu^*(z)\}$

$\therefore \mu^*(xyz) \geq \min\{\mu^*(x), \mu^*(z)\}$

Theorem:3.20

If μ is a fuzzy bi ideal of a near subtraction semigroup X, then $(\mu^*)^* = \mu^*$

Proof:

For any $x \in X$. We have

$(\mu^*)^* = \mu^*(x) + 1 - \mu^*(0)$

$=[\mu(x) + 1 - \mu(0)] + 1 - [\mu(0) + 1 - \mu(0)]$

$=[\mu(x) + 1 - \mu(0) + 1 - \mu(0) + 1 - \mu(0)]$

$= \mu(x) + 1 - \mu(0)$

$= \mu^*$

Therefore, $(\mu^*)^* = \mu^*$

Theorem:3.21

Let f: X → X' be a onto homomorphism of a near subtraction semigroup X. Then we have that

(1) If λ be a fuzzy bi-ideal of X', then $f^{-1}(\lambda)$ is a fuzzy bi-ideal in X.

(2) If μ be a fuzzy bi-ideal of X, then f(μ) is a fuzzy bi-ideal in X'.

Proof:

(1) Let λ be a fuzzy bi-ideal of X'.

Let $x, y, z \in X$.

(i) $f^{-1}(\lambda)(x-y) = \lambda(f(x-y))$

$= \lambda(f(x) - f(y))$

$\geq \min\{\lambda(f(x)), \lambda(f(y))\}$

$= \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\}$

$\therefore f^{-1}(\lambda)(x-y) \geq \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\}$

(ii) $f^{-1}(\lambda)(xyz) = \lambda(f(xyz))$

$= \lambda(f(x) f(y) f(z))$

$\geq \min\{\lambda(f(x)), \lambda(f(z))\}$

$= \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z)\}$

$\therefore f^{-1}(\lambda)(xyz) \geq \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z)\}$

Hence $f^{-1}(\lambda)$ is a fuzzy bi-ideal of X.

(2) Let μ be a fuzzy bi-ideal of X.

Let $y_1, y_2, y_3 \in X'$. Then we have

$\{x / x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2 / x_1 \in f^{-1}(y_1) \;\&\; x_2 \in f^{-1}(y_2)\}$

(i) $f(\mu)(y_1 - y_2) = \text{Sup}\{\mu(x) / x \in f^{-1}(y_1 - y_2)\}$

$\geq \text{Sup}\{\mu(x_1 - x_2) / x_1 \in f^{-1}(y_1) \;\&\; x_2 \in f^{-1}(y_2)\}$

$\geq \text{Sup}\{\min\{\mu(x_1), \mu(x_2) / x_1 \in f^{-1}(y_1) \;\&\; x_2 \in f^{-1}(y_2)\}$

$\geq \min\{\text{Sup}\{\mu(x_1) / x_1 \in f^{-1}(y_1)\} \;\&\; \text{Sup}\{\mu(x_2) / x_2 \in f^{-1}(y_2)\}$

$= \min\{f(\mu)(y_1), f(\mu)(y_2)\}$

Therefore $f(\mu)(y_1 - y_2) \geq \min\{f(\mu)(y_1), f(\mu)(y_2)\}$

Let $y_1, y_2, y_3 \in X'$.

(i) $f(\mu)(y_1 y_2 y_3) = \text{Sup}\{\mu(x) / x \in f^{-1}(y_1 y_2 y_3)\}$

$\geq \text{Sup}\{\mu(x_1 x_2 x_3) / x_1 \in f^{-1}(y_1) \;\&\; x_3 \in f^{-1}(y_3)\}$

$\geq \text{Sup}\{\min\{\mu(x_1), \mu(x_3) / x_1 \in f^{-1}(y_1) \;\&\; x_3 \in f^{-1}(y_3)\}$

$\geq \min\{\text{Sup}\{\mu(x_1) / x_1 \in f^{-1}(y_1)\} \;\&\; \text{Sup}\{\mu(x_3) / x_2 \in f^{-1}(y_3)\}$

$= \min\{f(\mu)(y_1), f(\mu)(y_3)\}$

$\therefore f(\mu)(y_1 y_2 y_3) \geq \min f(\mu)(y_1), f(\mu)(y_3)\}$

Hence f(μ) is a fuzzy bi-ideal in X'.

Theorem:3.22

If μ is normal anti-fuzzy bi –ideal of a near subtraction semigroup X iff $\mu^* = \mu$.

Proof:

The sufficient part is obivious. To prove the necessary part, let us suppose that μ is normal anti-fuzzy bi –ideal of a near subtraction semigroup X. Let $x \in X$. Since μ is normal.

$\mu^*(x) = \mu(x) + 1 - \mu(0)$

$= \mu(x) + 1 - 1$

$= \mu(x)$

Hence $\mu^* = \mu$

Theorem:3.23

Let μ be an anti-fuzzy bi-ideal of a near subtraction semigroup X, and t be fixed element of X such that $\mu(0) \neq \mu(t)$. Define a fuzzy set μ^* in X by

$\mu^*(x) = \frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}$ for all $x \in X$. Then μ^* is normal anti-fuzzy bi-ideal of a near subtraction semigroup X.

Proof:

Let μ be an anti-fuzzy bi-ideal of a near subtraction semigroup X.

For any $x, y \in X$.

(i) $\mu^*(x - y) = \frac{\mu(x-y) - \mu(t)}{\mu(0) - \mu(t)}$

$\leq \frac{\max\{\mu(x), \mu(y)\} - \mu(t)}{\mu(0) - \mu(t)}$

$= \max\left\{\frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}, \frac{\mu(y) - \mu(t)}{\mu(0) - \mu(t)}\right\}$

$= \max\{\mu^*(x), \mu^*(y)\}$

Therefore $\mu^*(x - y) \geq \max\{\mu^*(x), \mu^*(y)\}$

For any x , y , z $\in X$.

$\mu^*(xyz) = \frac{\mu(xyz) - \mu(t)}{\mu(0) - \mu(t)}$

$\leq \frac{\max\{\mu(x), \mu(z)\} - \mu(t)}{\mu(0) - \mu(t)}$

$= \max\left\{\frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}, \frac{\mu(z) - \mu(t)}{\mu(0) - \mu(t)}\right\}$

$= \max\{\mu^*(x), \mu^*(z)\}$

Therefore $\mu^*(xyz) \geq \max\{\mu^*(x), \mu^*(z)\}$

Hence μ^* is an anti- fuzzy bi-ideal of X.

Also $\mu^*(0) = \frac{\mu(0) - \mu(t)}{\mu(0) - \mu(t)} = 1$, μ^* is normal.

Since $t \in X$ and $\mu^*(t) = \frac{\mu(t) - \mu(t)}{\mu(0) - \mu(t)} = 0$

We have μ^* is a complete anti-fuzzy bi-ideal on X.

Theorem:3.24

Let μ be an anti-fuzzy bi-ideal of a near subtraction semigroup X, and let

f: $[0, \mu(0)] \rightarrow [0, 1]$ be an increasing function. Then the fuzzy set

$\mu_f(x) = f(\mu(x))$ is a anti-fuzzy bi-ideal of X. In particular,if $f[\mu(0)] = 1$ then μ_f is normal and if $f(t) \geq t$ for all $t \in [0, \mu(0)]$ then $\mu \subseteq \mu_f$.

Proof:

For any $x, y \in X$.

(i)$\mu_f(x - y) = f(\mu(x - y))$

$\leq f(\max\{\mu(x), \mu(y)\})$

$= \max\{f(\mu(x)), f(\mu(y))\}$

$= \max\{\mu_f(x), \mu_f(y)\}$

$\therefore \mu_f(x - y) \leq \max\{\mu_f(x), \mu_f(y)\}$

For any $x, y, z \in X$.

(ii)$\mu_f(xyz) = f(\mu(xyz))$

$\leq f(\max\{\mu(x), \mu(z)\})$

$= \max\{f(\mu(x)), f(\mu(z))\}$

$= \max\{\mu_f(x), \mu_f(z)\}$

$\therefore \mu_f(xyz) \leq \max\{\mu_f(x), \mu_f(z)\}$

Hence μ_f is a anti-fuzzy bi-ideal of X.

If $f[\mu(0)] = 1$ then $\mu_f(0) = 1$.

Thus μ_f is normal .Assume that

$f(t) = f[\mu(x)] \geq \mu(x)$, for any $x \in X$ which implies $\mu \subseteq \mu_f$

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