Fuzzy strong bi-ideal of near subtraction semigroups

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Abstract: **In this paper we introduce the notation of fuzzy strong bi-ideal of a near-subtraction semigroup and obtain a characterization of a strong bi-ideal in terms of a fuzzy strong bi-ideal of a near-subtraction semigroup. We establish that every fuzzy left X-subgroup fuzzy left ideal of near- subtraction semigroup is a fuzzy strong bi-ideal of a near-subtraction semigroup. But the converse is not necessarily true as shown by an example. Further, we discuss the properties of fuzzy strong bi-ideal of a near subtraction semigroup.**

Keywords: **Fuzzy two sided X-subalgebra, fuzzy subnear-subtraction semigroup, fuzzy bi-ideal, fuzzy strong bi-ideal.**

1. Introduction

B.M. Schein [6] considered systems of the form (X; o ;/), where X is a set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of [2]). Y.B.Jun et al [3] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals. In[3], Y.B.Jun and H.S.Kim established the ideal generated by a set,and discussed related results. The concept of fuzzy set was first initiated by Zadeh[7]. Narayanan et al.[5] defined the concept of generalized fuzzy ideals of near-rings. Mahalakshmi et al. [3] studied the notation of bi-ideals in near subtraction semigroups. Manikandan [4] studied fuzzy fuzzy bi-ideals in near-rings.

2. Preliminaries

Definition:2.1

A nonempty set X together with two binary operation – and. is called near subtraction algebra if it satisfying the following:

(i) x-(y-x) = x
(ii) x-(x-y) = y-(y-x)
(iii) (x-y)-z = (x-z)-y

Definition:2.2

A nonempty set X together with two binary operation – and . is said to be subtraction semigroup if it satisfying the following:

(i) (x,-) is a subtraction algebra.
(ii) (x,.) is a semigroup.
(iii) x(y-z) = xy-xz and (x-y)z=xz-yz ∀x,y,z□X

Note: 2.3

1. Let X be a near- subtraction semigroup. Given two subsets A and B of X,
A*B = {ab/a□A, b□B}. Also we define another operation "□□"
A□B = {a (b+i) – ab /a,b□A, i□B}.

Definition: 2.4

A near- subtraction semigroup X is called zero-symmetric, if x0 = 0, for all x in X.

Definition: 2.5

A Subtraction semigroup X is said to be **regular** if given a□X, there is x□□X such that axa =a.

Definition: 2.6

A near- subtraction semigroup X is said to be **left permutable** near-Subtraction semigroup if abc = acb, for all a,b,c in X.

Definition: 2.7

A function A from a non-empty set X to the **unit interval**.[0,1] is called a fuzzy subset of X.[14]

Notation: 2.8

Let A and B be two fuzzy subsets of a semigroup X. We define the relation □□between A and B, the intersection and product of A and B, respectively as follows:

1. A □□B if A(x) □□B(x), for all x□X,
2. (A□B) (x) = min{A(x), B(x)}, For all x□X
3. (A□B) (x) = $\begin{cases} \sup_{x=yz}\{min\{a(y), b(z)\} & \text{if } x=yz, \text{for all } y,z□ X \\ 0 & \text{Otherwise} \end{cases}$

It is easily verified that the "product" of fuzzy subsets is associative. Throughout this paper, X will denote a near-subtraction semigroup unless otherwise specified. We denote by k_I the characteristic function of a subset I of X. The characteristic function of X is denoted by **X**, that is, **X**: X□[0,1] mapping every element of X to 1.

Definition: 2.9

A fuzzy subset A of X is said to be a fuzzy ideal of X if

1. A is a fuzzy near-subtraction semigroup of X,
2. A(y+x-y) = A(x), for all x, y□X,
3. A(xy) □□A(x), for all x,y□X,
4. A(ai – a(b-i) □□A(i), for all a,b, i,□X.

If A satisfies (1) and (2) and (3) then A is called a fuzzy right ideal of X. If A satisfies (1), (2) and (4), then A is called a fuzzy left ideal of X. In case of zero-symmetric, If A satisfies (1),(2) and A(xy) □□A(y), for all x, y□X and A is called a fuzzy left ideal of X.

3. Fuzzy Strong Bi-ideals of Near- subtraction semigroup

Definition: 3.1

A fuzzy bi-ideal A of X is called a fuzzy strong bi-ideal of X, if X□A□A □□A

Example: 3.2

Let X={0,a,b,c} be a near-subtraction semigroup with two binary operations „□" and „●" is defined as follows.

-	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

●	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Define a fuzzy subset A: X □[0,1] by A(0) = 0.8, A(a) = 0.3, A(b) = 0.6,
A(c) = 0.3.ThenX□A□A(0)= 0.3,
X□A□A(a) =0 0, X□A□A (b) = 0,
X□A□A (c) =0, and so A is a fuzzy strong bi-ideal of X

Note: 3.3

Every fuzzy strong bi-ideal is fuzzy bi-ideal. But the converse is not true.

Example: 3.4

Let X={0,a,b,c} be a near-subtraction semigroup with two binary operations „□" and „□" is defined as follows..

-	0	b	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

●	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a0	a	c	c

Define a fuzzy subset A: X□□[0,1] by
A(0) = 0.9, A(a) = 0.4, A(b) = 0.4,
A(c)=0.7,Then(**A**□X□A)(0)=0.9,
(**A**□X□A)(a)=0.7,(**A**□X□A)(b)=0.4, (**A**□X□A)(c) = 0.7,((**A**□X)*A)(0)=0.9,
((**A**□X)*A)(a)=0,((**A**□X)*A)(b)=0.7, ((**A**□X)*A)(c)=0,
X□A□A(0)=0.3,
X□A□A(a) =0,**X**□A□A(b) = 0.7, **X**□A□A(c)=0. Then A is a fuzzy bi-ideal of X. But not a fuzzy strong bi-ideal,since
X□A□A (b)$\not\leq A$(b)

Theorem: 3.5

Let {Ai □□i□J} be any family of fuzzy strong bi-ideals of X. Then A =$\bigcap_{i\in J} A_i$□is a fuzzy strong bi-ideal of X, where J be an index set. iεJ

Proof

By Theorem 3.4,[4] A is a fuzzy bi-ideal of X. Now for all x□X, since
A =□$\bigcap_{i\in J} A_i$□□□Ai for every i□J, we have
(**X**□A□A) (x) ≤ (**X**□Ai□Ai) (x)
≤ Ai(x) for every i□J
(Since Ai is a fuzzy strong bi-ideal of X)
It follows that,
(**X**□A□A) (x) ≤ inf{ Ai(x) : i□J}
=$\bigcap_{i\in J} A_i$(x)
= A(x)
Thus **X**□A□A □□A. So A is a fuzzy strong bi-ideal of X.

Theorem: 3.6

Let I be a non-empty subset of X and KI be a fuzzy subset of X. Then the following
conditions are equivalent:
1. I is a strong bi-ideal of X.
2. KI is a fuzzy strong bi-ideal of X.

Proof

First assume that I is a strong bi-ideal of X. Then I is a bi-ideal of X. By Theorem 3.8[4] , weget KI is a fuzzy bi-ideal of X.
Let a be any element of X. If a□I then KI(a) = 1 ≥ (X□KI□KI) (a). If a□I then
KI(a)=0. On the other hand assume that(**X**□KI□KI)(a)=1 Then
(X□K_I□K_I)(a)=$\sup_{a=pq} min${X□□□□p),K_I(q)}

=$\sup_{a=pq} min\{\sup_{p=p_1p_2}\{X(p_1), k_1(p_2\}, k_1(q)\}$
(since **X**(x) = 1 , □□x□X)

=$\sup_{a=pq} min\{\sup_{p=p_1p_2} k_1(p_2\}, k_1(q)\}$=1

And $k_1(p_2)$=1,$k_1(q) = 1$.So p_2 , q□□.
Then a = pq =p_1p_2 q □□XII □□I which contradictsa□I.ThusKI (a)=0=(**X**□KI□KI)(a). This shows that
(**X**□KI□KI) □□□□KI. Therefore KI is a fuzzy strong bi-ideal of X.
Conversely, assume that KI is a fuzzy strong bi-ideal of X. Every fuzzy strong bi-ideal of X is a fuzzy bi-ideal of X. Therefore by Theorem 3.8,[4] I is a fuzzy bi-ideal of X. Let a be any element of XI^2. Then there exists a,p,q,p_1 of X and the elementsp_1, q of I such that a = pq and
p = p_1 p_2.
(X□K_I□K_I)(a)=$\sup_{a=pq} min${X□□□□p),□□□q)}

= $\sup_{a=pq} min\{\sup_{p=p_1 p_2} min\{X(P_1), k_1(p_2), k_1(q)\}$

= $\sup_{a=pq} min\{\sup_{p=P_1P_2} min, \{k_1(p_2)\}, k_1(q)\}$
= min {1,1}=1
(KI) (a) ≥ (**X**□KI□□KI) (a) = 1. Thus a□I.So XII □□I. This shows that I is a strong bi-ideal of X.

Theorem: 3.7

Every left permutable fuzzy right
X-subgroup of X is a fuzzy strong bi-ideal of X.

Proof

Let A be a left permutable fuzzy right
X-subgroup of X.
To prove A is a fuzzy strong bi-ideal of X.
By Theorem 3.9,[4] we get every fuzzy right X-subgroup of X is a fuzzy bi-ideal of X. Choosea, b,c, b_1,b_2□X such that
a = bc and b = b_1, b_2 . Then
X□A□A (a) =$\sup_{a=bc} min${X□□□b□□□□□c□□

=$\sup_{a=bc} min\{\sup_{b=b_1b_2} min\{X(b_1), A(b_2)\}, A(c)\}$

= $\sup_{a=bc} min$ {$\sup_{b=b_1b_2}$min{ A(b_2),A(c)}
(Since A is a left permutable fuzzy right X-subgroup of X,
A(bc) = A((b1b2)c) = A((b2b1) c)□A(b2)) and **X**(c) □A(c)
□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
□□□□□□□□$\sup_{a=bc} min${□□bc□□□X□c□□

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$\sup_{a=bc} min${□□bc□□□□□

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Therefore **X**□A□A □□A. Hence A is a fuzzy strong bi-ideal of X.

Theorem: 3.8

Every fuzzy left X-subgroup of X is a fuzzy strong bi-ideal of X.

Proof

Let A be a fuzzy left X-subgroup of X.

To prove A is a fuzzy strong bi-ideal of X.

By Theorem 3.10,[4] we get every fuzzy left X-subgroup of X is a fuzzy bi-ideal of X. Choose a, b,c,c_1, c_2□X such that a = bc and c =c_1, c_2 . Then

X□A□A (a) =□ $\sup_{a=bc} min${X□b)□□A□A□c□□

= $\sup_{a=bc} min${X(b),$\sup_{c=c_1c_2} min\{A(c_1)\}, A(c_2)\}$

=$\sup_{a=bc} min\{1, \sup_{c=c_1c_2} min\{A(c_1)\}, A(c_2)\}$

(Since A□is a fuzzy left X-subgroup of X, A (bc)=A(b(c_1c_2))=A((bc_1)c_2)□A(c_2))

□□□□□$\sup_{a=bc} min${X(c_1),□□bc□□

=$\sup_{a=bc} min${1,□□bc□□

=A(bc)

= A(a)

Therefore **X**□A□A □□A. Hence A is a fuzzy strong bi-ideal of X.

Theorem: 3.9

Every left permutable fuzzy two-sided X-subgroup of X is a fuzzy strong bi-ideal of X.

Proof

The proof is straight forward from the above Theorem 3.5 and Theorem 3.6

Theorem: 3.10

Every left permutable fuzzy right ideal of X is a fuzzy strong bi-ideal of X.

Proof

Let A be a left permutable fuzzy right ideal of X.

To prove A is a fuzzy strong bi-ideal of X.

By Theorem 3.12,[4] we get every fuzzy right ideal of X is a fuzzy bi-ideal of X. Choose a,b,c,b_1,b_2□X such that a = bc and b=b_1,b_2 Then

X□A□A (a)= $\sup_{a=bc} min${X□A(b),□□c□□

□□□$\sup_{a=bc} min\{ \sup_{b=b_1b_2} min\{X(b_1), A(b_2)\}, A(c)\}$

=$\sup_{a=bc} min\{ \sup_{b=b_1b_2} min\{A(b_2)\}, A(c)\}$

(Since A is a left permutable fuzzy right ideal of X,A(bc) =A((b1b2)c) =A((b2b1)c) A(b2(b1)c)□A(b2))and **X**(c) □A(c)

□□□ $\sup_{a=bc} min${A(bc),X□c□□

= $\sup_{a=bc} min${A(bc),1□

□□□□□□A(a)

Therefore **X**□A□A □□A. Hence A is a fuzzy strong bi-ideal of X.

Theorem: 3.11

Every fuzzy left ideal of X is a fuzzy strong bi-ideal of X.

Proof

Let A be a fuzzy left ideal of X. To prove A is a fuzzy strong bi-ideal of X.

By Theorem 3.13[4] we get every fuzzy left ideal of X is a fuzzy bi-ideal of X. Choose a,b,c,b_1,b_2□X such that a = bc = bi-b(c-i) Then

X□A□A (a)= $\sup_{a=bc} min${ X□A(b)□□A□c□□

= □$\sup_{a=bc} min\{ \sup_{b=b_1b_2} min\{X(b_1), A(b_2)\}, A(c)\}$

= $\sup_{a=bc} min\{ \sup_{b=b_1b_2} min\{A(b_2)\}, A(c)\}$

(Since A is a fuzzy left ideal of X, A(bi- b(c-i)) ≥A(c) and X(b_2) ≥ A(b_2)

≤ $\sup_{a=bc} min${X(b_2), A(bi- b(c-i))□

□□□□□□□ $\sup_{a=bc}$ {A(bi- b(c-i))□

□□□□□□□□□□□□□bc)

□□□□□□□□□□□□□□a□

Therefore **X**□A□A □□A. Hence A is a fuzzy strong bi-ideal of X.

Theorem: 3.12

Every left permutable fuzzy ideal of X is a fuzzy strong bi-ideal of X.

Proof:

The proof is straight forward from 3.8 and Theorem 3.9 the Theorem

Remark: 3.13

The converse of Theorem 3.7 and Theorem 3.10 are not necessarily true as shown by the following example.

Example: 3.14

Let X={0,a,b,c} be the near Subtraction semigroups with two binary operations „□" and „□" is defined as follows.

Define a fuzzy subset A: X□□[0,1] by

A (0) = 0.75, A(a) = 0.2, A(b) = 0.3,

A(c) =0.3.Then (**A**□X□A)(0)=0.3,

(**A**□X□A) (a) =0, (**A**□X□A)(b) = 0,

(**A**□X□A) (c) = 0, **X**□A□A (0) = 0.3,

X□A□A (a) = 0, **X**□A□A (b)=0,**X**□A□A (c) = 0, and so A is a fuzzy strong bi-ideal of X. Since A (a) = A (bc) ≱A(b)and A (a) = A (bc) ≱ A(c), A is not a fuzzy two-sided X-subgroup of X.

Since A(a) =A(bc)≥ min {A(b),A(c)}, A is not a fuzzy sub near-ring of X and so A is not a fuzzy ideal of X.

-	0	a	b	c		□	0	a	b	c
0	0	0	0	0		0	0	0	0	0
b	a	0	a	0		a	0	0	0	0
b	b	b	0	a		b	0	0	0	a
c	c	b	a	0		c	00	0	0	a

Theorem: 3.15

Let A be any fuzzy strong bi-ideal of a near-Subtraction semigroups X. Then

A(axy) □□min{ A(x), A(y)} □a, x, y□X.

Proof

Assume that A is a fuzzy strong bi-ideal of X. Then **X**□A□A □□A.

Let a, x and y be any element of X. Then

A(axy) □□(**X**□A□A**)** (axy)

= $\sup_{axy=pq} min${ X□A(p),A(q)□

□□□□□□□□□□□□□□□□□□min{ X□A (ax),A(y)□

□□= $min\{ \sup_{ax=z_1z_2} min\{X(z_1), A(z_2)\}, A(y)\}$

= □ $min\{ min\{X(a), A(x)\}, A(y)\}$

= □ $min\{min\{1, A(x)\}, A(y)\}$

= □ $min\{ A(x), A(y) \}$

□This shows that

A(axy) □□min{ A(x), A(y)} □a, x, y □X.

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